# MATH 155 - Chapter 8.8 - Improper Integrals

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1. **Definition of Improper Integrals with Infinite Integration Limits:**

   1. If $f$ is continuous on $[a, \infty)$, then
   $$\int_a^\infty f(x)\,dx = \lim_{k\to\infty} \int_a^k f(x)\,dx$$
   2. If $f$ is continuous on $(-\infty, b]$, then
   $$\int_{-\infty}^b f(x)\,dx = \lim_{k\to-\infty} \int_k^b f(x)\,dx$$
   3. If $f$ is continuous on $(-\infty, \infty)$ and $c$ is any real number, then
   $$\int_{-\infty}^\infty f(x)\,dx = \lim_{k\to-\infty} \int_k^c f(x)\,dx + \lim_{j\to\infty} \int_c^j f(x)\,dx$$

   We say that the improper integral **converges** if the limit exists (the limit is a finite number). We say that the improper integral **diverges** if the limit does not exit (the limit goes to $\pm\infty$).

2. **Definition of Improper Integrals with Infinite Discontinuities:**

   1. If $f$ is continuous on $[a, b)$ and has an infinite discontinuity at $b$, then
   $$\int_a^b f(x)\,dx = \lim_{k\to b^-} \int_a^k f(x)\,dx$$
   2. If $f$ is continuous on $(a, b]$ and has an infinite discontinuity at $a$, then
   $$\int_a^b f(x)\,dx = \lim_{k\to a^+} \int_k^b f(x)\,dx$$
   1. If $f$ is continuous on $[a, b]$, except for some $c$ in $(a, b)$ at which $f$ has an infinite discontinuity, then
   $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \lim_{k\to c^-} \int_a^k f(x)\,dx + \lim_{j\to c^+} \int_j^b f(x)\,dx$$

3. **Theorem: A Special Type of Improper Integral**

$$\int_1^\infty \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1} & \text{if } p > 1 \\ \text{diverges} & \text{if } p \leq 1 \end{cases}$$